

## Conclusions

In the present work, a methodology for three-dimensional structured multiblock grid generation in axial turbomachinery was presented. There is a main O grid around each blade, constructed by stacking two-dimensional blade-to-blade surface O grids generated biharmonically. Only a few user-defined parameters are needed for generating the blade-to-blade surface O grids, and experience has shown that the choice of these parameters is a quite straightforward task. Biharmonic grid generation ( $\nabla^4 \xi = 0$ ,  $\nabla^4 \eta = 0$ ) has the decisive advantage over elliptic grid generation ( $\nabla^2 \xi = Q_\xi$ ,  $\nabla^2 \eta = Q_\eta$ ) in that it allows automatic control of both position and orthogonality at the boundaries. The numerical implementation used ( $\nabla^2 \xi = Q_\xi$ ,  $\nabla^2 Q_\xi = 0$ ;  $\nabla^2 \eta = Q_\eta$ ,  $\nabla^2 Q_\eta = 0$ ) clearly shows that biharmonic grid generation computes the source terms  $Q_\xi$  and  $Q_\eta$ , which are user-defined in Poisson grid generation. The biharmonicsolver gives a quasiorthogonal initial grid that is not yet refined near the solid walls. Stretching to accurately resolve boundary layers is obtained by interpolating points on computed grid lines.

Tip-clearance gaps are meshed using O-type grids, generated biharmonically. The radial surfaces of the main blade-to-bladesurface O grid are stretched near the casing and the hub. The clearance-gap TC grid is stretched both at the casing and at the blade tip. The radial distribution of the TC grid is independent of the radial distribution of the O grid. A buffer OZ grid that overlaps with the O grid, and that is stretched both at the casing and at the blade tip, is used to accurately resolve the flow coming over the tip.

The method is quite robust and provides good-quality structured grids. It has been applied to the computation of various compressor and turbine configurations using a three-dimensional Navier–Stokes solver with multiequation turbulence closures.<sup>3,9</sup> In the case of starters with clearance gap at the hub, grid generation is done in an exactly analogous way.

## Appendix: Geometric Stretching

The geometric stretching used stretches  $N_1$  of the  $N$  points near the starting location [ $s_1 = s(1) = 0$ ] with ratio  $r_1$ , stretches  $N_2$  points near the final location [ $s_2 = s(N) = L$ ] with ratio  $r_2$ , and places  $N - N_1 - N_2$  equidistant points in between:

$$l_{gs}(n, L, N, N_1, r_1, N_2, r_2) = \begin{cases} s(1) + \frac{r_1^{n-1}-1}{r_1-1}hr_1^{-(N_1-2)}, & n = 2, \dots, N_1 \\ s(N_1) + (n - N_1)h, & n = N_1 + 1, \dots, N - N_2 + 1 \\ s(N - N_2 + 1) + \frac{r_2^{-(n-N+N_2-1)}-1}{r_2^{-1}-1}h, & n = N - N_2 + 2, \dots, N \end{cases} \quad (A1)$$

$$h = L \left[ N - N_1 - N_2 + 1 + \frac{r_1^{-(N_1-1)}-1}{r_1^{-1}-1} + \frac{r_2^{-(N_2-1)}-1}{r_2^{-1}-1} \right]^{-1} \quad (A2)$$

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# Adaptive Analysis of Oscillating Cascade Flows on a Quadrilateral-Triangular Mesh

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## Introduction

IN recent years, a number of Euler<sup>1,2</sup> and Navier–Stokes<sup>3,4</sup> solvers have been presented to simulate the blade vibration problems. Wolff and Fleeter<sup>1</sup> applied the Fourier series lagged boundary condition treatment on an expanded grid along the periodic boundary. Hwang and Yang<sup>2</sup> presented a rigid-deformable dynamic mesh algorithm to investigate transonic flows around an oscillating cascade of four blades. On a multipassage computational mesh,<sup>3</sup> the explicit four-stage Runge–Kutta scheme and the Baldwin–Lomax mixing-length turbulence model were adopted. The calculation showed that there was a more apparent mesh dependence of the results in the regions of flow separation. Within a composite grid where a deforming C-grid was embedded in an H-grid, a coupled inviscid/

viscous model<sup>4</sup> was implemented to incorporate the inverse integral boundary-layersolution and the time-marching NPHASE analysis. The purpose of this work is to present a solution-adaptive solver to investigate the transonic oscillating cascade flows on a quadrilateral-triangular mesh. The Euler equations with moving domain effects are solved in the Cartesian coordinate. This solver

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includes the locally implicit total variation diminishing scheme, dynamic mesh algorithm,<sup>2</sup> interpolation method, two-level refinement procedure,<sup>5</sup> and enrichment indicator.

### Adaptive-Mesh Technique

The present adaptive-mesh technique includes the mesh enrichment indicator, two-level refinement procedure, and interpolation method. In the present calculations, the absolute value of the substantial derivative of Mach number  $|DM/Dt|$  is chosen as the mesh enrichment indicator. As for the two-level refinement procedure, the mesh enrichment is operated on the background grid (initial grid) instead of the last adapted mesh. Initially, the value of  $|DM/Dt|$  of each unrefined cell is calculated. The product of a specified constant  $C_1$  and the average value of  $|DM/Dt|$  over the entire background grid is selected as the first threshold value. If the value of  $|DM/Dt|$  of each unrefined cell is larger than the first threshold value  $C_1 * |DM/Dt|_{av}$ , the new node will be placed at the midpoint of each edge of the quadrilateral-triangular cell or the center of the quadrilateral cell.<sup>5</sup> After finishing the first-level refinement, the properties at all added new nodes are interpolated from those at the background nodes. Continuing the second-level mesh refinement, the value of  $|DM/Dt|$  for each cell on the intermediate mesh and the corresponding second threshold value  $C_2 * |DM/Dt|_{av}$  are computed. Then the intermediate mesh is refined by processing the first-level refinement technique.

### Results and Discussion

In the present calculations, the inlet Mach number and exit pressure ratio are set to be 0.8 and 0.7322, respectively. The computational domain contains four uncambered biconvex blades, where the values of thickness-to-chord ratio, solidity, and stagger angle are 0.076, 1.3, and 53 deg, respectively. The motion of these four blades, which is executing torsional mode oscillations about midchord, is governed by the following:

$$\alpha = \alpha_0 + \alpha_l \sin(2Mkv + m\sigma) \quad (1)$$

where  $\alpha$ ,  $\alpha_0$ ,  $\alpha_l$ ,  $M$ ,  $k$ ,  $v$ , and  $\sigma$  represent the instantaneous angle of attack, mean flow angle of attack, oscillation amplitude, inlet Mach number, reduced frequency, and nondimensionalized time scale and interblade phase angle, respectively. The blade numbers  $m = 0, 1,$

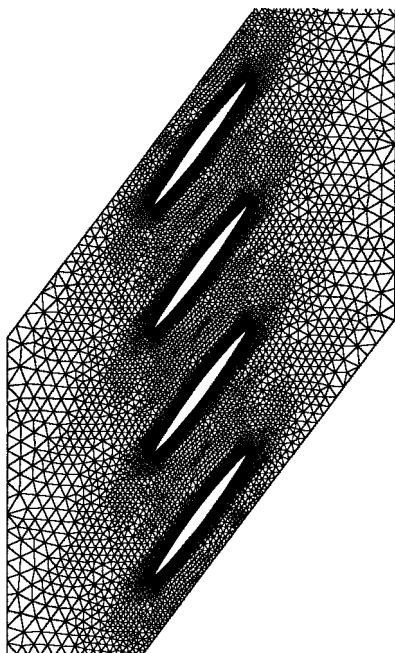


Fig. 1 Partial view of the quadrilateral-triangular mesh (13,830 elements and 9619 nodes).

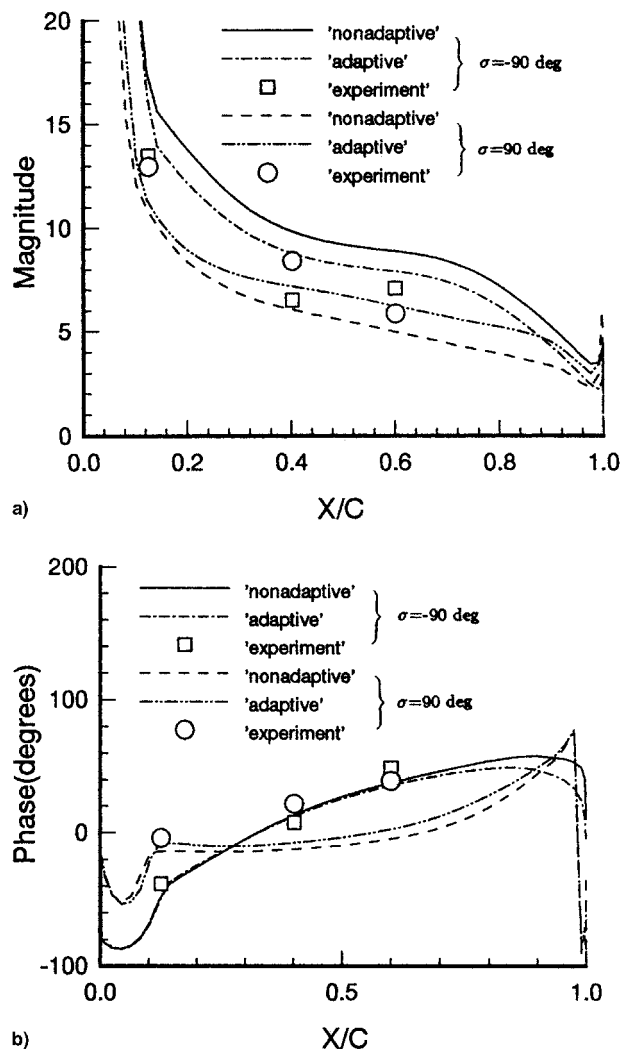


Fig. 2 a) Magnitude and b) phase angle of the first harmonic dynamic pressure difference coefficient  $\Delta C_p$  for an oscillating cascade of four blades ( $k = 0.462$ ,  $\alpha_0 = 7$  deg, and  $\alpha_l = 1.2$  deg).

2, and 3 represent each blade from the lowest to the highest one, respectively. As shown in Fig. 1, the quadrilateral-triangular mesh contains 13,830 elements and 9619 nodes. The periodic solution is achieved by processing six cycles of motion, and it takes 3600 time steps to accomplish one cycle of motion.  $C_1$  and  $C_2$  are set to be 0.5 and 2.5, respectively, and adaptation is performed every 36 time steps. If adaptation is performed every 15 time steps, an identical result will be obtained. The present solution is calculated on a Dec-3000 workstation, and it takes 13.7 computational seconds per iteration.

When one value of  $\alpha_l$  (1.2 deg) and two values of  $\sigma$  ( $-90$  and  $90$  deg) are chosen, magnitudes and phase angles of the first harmonic dynamic surface pressure difference coefficient  $\Delta C_p$  are calculated and plotted in Fig. 2. To evaluate the present adaptive solver, the experimental data<sup>6</sup> and Euler solutions obtained on the nonadaptive quadrilateral-triangular mesh<sup>2</sup> are adopted for comparison. By choosing the experimental data as the reference values, the distributions of magnitude in Fig. 2a indicate that the present adaptive solver provides better results than does the nonadaptive approach. For the phase-angle distributions with  $\sigma$  equal to  $90$  deg in Fig. 2b, the same conclusion is drawn. When the value of  $\sigma$  is replaced by  $-90$  deg, the difference between adaptive and nonadaptive results is slight, and both solutions compare well with the experimental data.

As for the case with  $\sigma$  and  $\alpha_l$  equal to  $90$  and  $4.8$  deg, respectively, the instantaneous mesh and Mach number contour ( $2Mkv = 12\pi$ )

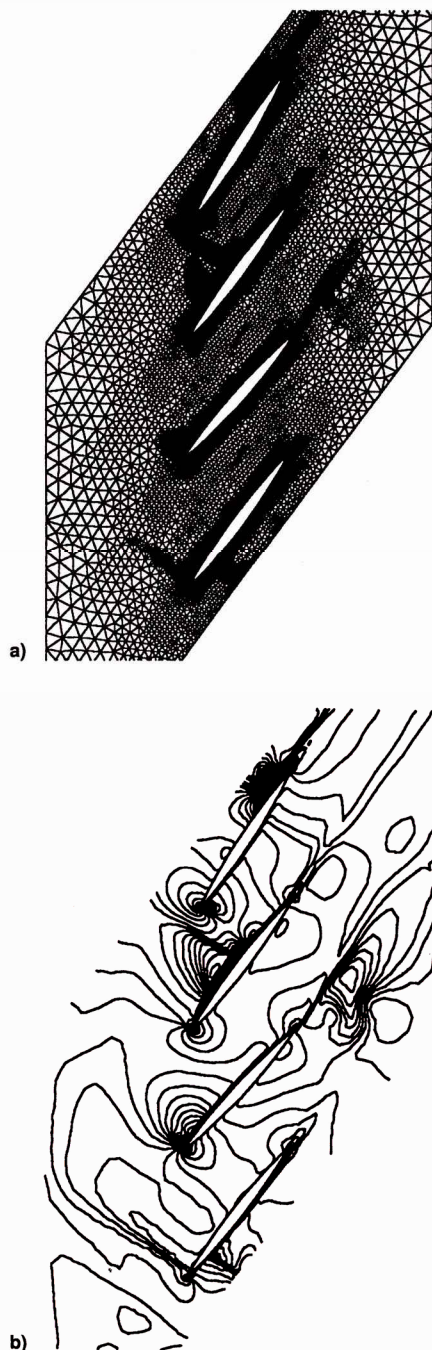


Fig. 3 a) Instantaneous mesh and b) Mach number contour for an oscillating cascade of four blades ( $k = 0.462$ ,  $\alpha_0 = 7$  deg,  $\alpha_I = 4.8$  deg,  $\sigma = 90$  deg, and  $2Mk\tau = 12\pi$ ).

are plotted in Fig. 3. From the results given in Fig. 3, shocks appear on the front part of the upper surface of the second blade, and on the midchord of the upper surface of the third blade. Meanwhile, a compression wave on the upper leading edge of the lowest blade is observed. Within the passage between the lowest and highest blades, a shock is depicted that is strong enough to choke the flowfield.

### Conclusions

In the present work, a solution-adaptive approach has been presented to investigate the transonic oscillating cascade flows. In a Cartesian coordinate system, the unsteady Euler equations are solved. Comparing the distributions of magnitude and phase angle

of the first harmonic dynamic pressure difference coefficient, the present adaptive solutions show better agreement with the experimental data than those from the nonadaptive approach. From the comparison and discussion of instantaneous mesh and Mach number, it is evident that the present adaptive mesh clearly captures the unsteady wave behavior.

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## Theoretical Prediction of Mean Droplet Size of Y-Jet Atomizers

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### Nomenclature

$C_d$	= discharge coefficient, nondimensional
$D_0$	= mixing chamber diameter, m
$d_l$	= ligament diameter, m
$f$	= constant, nondimensional
$h_0$	= liquid sheet thickness at the nozzle tip, m
$K$	= nozzle parameter, defined in Eq. (5), ms
$l_0$	= mixing chamber length, m
$m_{\text{air}}$	= atomizing airflow rate, kg s <sup>-1</sup>
$m_f$	= mass flow rate of liquid, kg s <sup>-1</sup>
$P_c$	= chamber pressure, Pa
$P_s$	= ambient pressure, Pa
$P_0$	= stagnation pressure, Pa

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